Chaotic Behavior of Financial Time Series-An Empirical Assessment

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Abstract

Financial Time Series often exhibit either chaotic or persistent or mean reversing behaviour. This behaviour could be quantified through Chaotic Exponent which ranges from zero to one. The rescaled range technique developed for hydrology by Hurst is applied in financial time series to estimate chaotic exponents which determines whether financial time series behaviour is purely chaotic white noise or any pattern exists. We have computed Chaotic Exponent coefficient for nine shares traded in Kula Lumpur Stock Exchange, nine popular stock market indices and nine selected exchange rates. As per theory pure random time series which behaves in a chaotic form cannot be forecasted, but somewhat skewed or non-normal financial time series could be forecasted. The forecasts could be used in several financial decisions like pricing of derivatives and very useful in hedging decisions. Our results indicate that the chaotic exponent of the selected financial time series is not consistent. They are either mean reversing or persistent, occasionally they show pure randomness. This proves that the forecasting of financial time series is relevant for hedging decisions.

Key words: Chaos, Brownian motion, Chaotic exponent, Financial time series, Persistent, Mean reversing

Introduction

The financing function which started in late fifties with net income and net operating income theories underwent several changes. Borrowing and value of firm, tested by researchers in the last few decades has gone to the background giving way for structured financial products and non linear exotic options whose pay offs are not at all linear. The financial engineers now design methodologies to price these products based on some expectations of return distributions. These engineers apply very strong assumptions while modeling returns for forecasting. One of the major assumptions is that the returns generated by any financial time series (FTS) whether share prices, exchange rates or market indices are normally distributed and they move at random. This assumption is disputed by many researchers (Aparicio et al., 1999, Gilmore 1993, Kyrtsou et al., 2004, Muckley 2004, Sewell et al., 1996, Varson et al., 1995). Risk management requires an accurate forecasting model to quantify the future price path of shares and other FTS. The forecasted price path is efficiently applied in managing the risks. Risk management involves managing the risk at minimum cost by choosing right risk mitigating instrument which include an array of financial instruments like forward contracts, futures contracts, option contracts and swaps.

These instruments' price movements are cointegrated with the prices of underlyings on which they issued and they closely follow the price path of the underlyings. Any deviation is quickly corrected and mostly they derive their prices from their underlying. The risks in financial instruments arise due to the ever fluctuating nature of their prices. Till last decade the loan interest rate was fixed over time but at present this has become variable. Good forecasting models are needed to forecast and plan these instruments' cash flows. Linear forecasting models like regression etc are out of pace. Modern models like ARIMA, GARCH and exponential models rely on normality and stationary assumptions. Many research studies point out deficiencies in forecasting models due to the strong assumptions on which they are built (Urrutia et al., 2002, 2006, Schittenkopf et al., 2000). Empirical studies which use the distributive models frequently fail due to the presence of various forms of distributions like normal, lognormal, t, exponential etc. There is no consensus among researchers regarding the character, structure and distributions these returns generate. Two popular distributions are applied as of date are the normal (also known as Gaussian or Weinar) and fat tail 't' distribution to model returns.

Lognormal distribution has become highly popular over the other distributions in modern financial engineering as the returns show non normality, nonlinearity and geometric growth characters (Baillie, 1989). After the arrival of Black Scholes model of option pricing the lognormal distribution has taken a center stage in financial engineering. Modeling of random movements in pollen particles started in 1820s by Brown and followed by Einstein and established by Weiner.

Brownian Motion

Brown a botanist studied the pollen particles movement suspended on water. He came to the conclusion that the pollen particles move randomly due to the innumerable bombardment of water molecules on it. Similar analogy is given for financial time series returns. Innumerable small trading transactions hit the prices constantly and hence the prices move randomly (Mouck, 1998). Brownian motion is purely random and any movement in time is purely stochastic and it does not depend on the previous motions or directions (Kyrtsou, 2004, Small, 2003). It is like dice throw in snake ladder game where the second throw's outcome does not depend on the first throw's result. The spaces are identically distributed in the dice. The distribution is Gaussian (normal) and the outcomes are purely random and there is no pattern. Similar analogy is applied in financial time series (FTS). The share prices, share market indices, interest rates and the exchange rates' generate returns which move at random and any movement in time is purely stochastic and it does not depend on the previous motions and there is no memory. Whatever oscillations noticed is pure white noise and no property, pattern or trend could be observed.

According to Markove process the past FTS returns cannot be used to predict the future returns or prices in any meaningful way as the FTS returns have no memory (Barkoulas, 1999, Cheung, Y.W. 1993, Lo A.W. 1991). The next price or return depends only on the previous value and not on earlier values. The above opinions are disputed by several researchers and they point out the FTS returns are not exactly Gaussian. These FTS returns show persistence behaviour (meaning that an increase in return is followed by a series of increases in return and vice versa). Some FTS returns show anti persistence or mean reversing behaviour (an increase in return will be followed by a series of decrease in returns). Many earlier research studies have pointed out that the FTS returns exhibit fat tails through serial correlations and run tests. The skewness and kurtosis measures are also utilised to support their view (Claire 2001, Fox 1986, Robinson 1995). They argue since the FTS returns are not exactly Gaussian therefore the prediction and forecasting models are relevant. They point out chartist approach and the derivative pricing etc to strengthen their argument. The Black and Scholes model and most of the modern financial asset price forecasting models assume Brownian motion and Weiner process in FTS returns and the returns of these FTS returns are normally distributed. Based on these strong assumptions the option premiums are calculated and the hedging decisions are made by the financial analysts, speculators and arbitrageurs.

Significance

It is therefore interesting to test what is the real position of FTS returns generated by shares, indices and exchange rates etc. The objective of this paper is to assess whether the FTS returns are purely white noise or any property or pattern exist. If shocks are pure white noise all forecasting models will become useless. We have attempted to assess whether the FTS returns show randomness or otherwise.Our argument goes in the following lines. The FTS returns will exhibit some pattern in the past and the same pattern or behavior will tend to continue in the future. All of a sudden FTS returns will not show altogether a new pattern unless there is an external random exogenous natural event or a disaster. The process of change will be gradual and mostly dependent on the nature of information which is disseminated. These patterns were assessed in the past by serial autocorrelations and run tests. The chaotic exponent (CE) is a new arrival in this domain to quantify the randomness or otherwise present in FTS returns (Kang 2004).The remaining part of this paper is organized into four sections. Section two explains the statistical background and section three explains the methodology adopted in this paper. The final section interprets the results and concludes the paper.

Chaotic Exponent (CE)

CE was originally applied in studying the river Nile's water flow pattern and now the same technique is applied in many branches of science including finance. The stochastic process (Brownian motion / Weiner process) present in FTS returns is quantified in CE. This exponent is computed by rescaled range (RSR) technique (Booth 1982). This method applies the natural log-time as independent variable and natural log-FTS returns as dependent variable in power law framework to compute the CE either through regression or through loglog plot.

The CE value lies in the range of $0 < C < 1$. An exponent in the range of 0.45 to 0.55 is interpreted as a pure stochastic random movement and whatever fluctuations noticed in FTS returns are a pure white noise and there is no trend or pattern. In other words it is a pure Brownian motion and exhibits a Gaussian distribution. Planning, forecasting, hedging and controlling are meaningless when the FTS returns are a pure white noise. If the CE is less than 0.45, it is interpreted as anti-persistence which means a price fall will be in all probabilities will follow price increases for sometime in the future. Technically it is described as mean reversing process. CE of more than 0.55 indicates persistence behaviour meaning a price increase will be followed by a few price increases in all probabilities and vice versa for sometime in the near future.

Methodology

The prices pertaining to share, indices and exchange rates are provided in the column vector form, P_i to calculate the CE where i = 1, 2 . . . N. N is the length of the vector. Financial literature is skeptical about the above vector of FTS because the FTS prices are non stationary meaning that they have no stable mean and variance. Hence the FTS prices are detrended to get returns as these returns are stable in mean and variance. Therefore the returns are important than the FTS prices in modeling. The change in prices is the returns and they are computed in several ways but all show almost similar results.

$$r_{t-1,t} = p_t - p_{t-1} \qquad (1)$$

$r_{t-1,t}$ = *pure returns*

$$r_{t-1,t} = \frac{p_t}{p_{t-1}} - 1 \qquad (2)$$

$r_{t-1,t}$ = *relative returns*

$$r_{t-1,t} = \ln p_t - \ln p_{t-1} \qquad (3)$$

$r_{t-1,t}$ = *geometric returns*

where p = Price, t = Time in days, ln = Natural logarithm

From the vector of share returns the mean and standard deviation are computed as follows.

$$\zeta = \frac{1}{n}\sum_{i=1}^{n} r_i \qquad (4)$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(r_i - \zeta)^2} \qquad (5)$$

Where ζ = mean returns, σ = standard deviation of returns, n = number of returns

FTS return deviations are computed with the above mean

$$d_i = r_i - \zeta \qquad (6)$$

Where, d = deviation (shock)

The individual deviations (shocks) are added consecutively to get cumulative deviations vector whose values will show the cumulative noise present in FTS at every stage. Both positive and negative shocks are added to get cumulative vector of deviations.

$$cd_1 = d_1$$
$$cd_2 = cd_1 + d_2$$
$$cd_3 = cd_2 + d_3$$
$$cd_n = cd_{n-1} + d_n \qquad (7)$$

where , cd_n = cumulative deviations

The cumulative return vector will not grow continuously as there are positive and negative returns. In the case of more positive returns the cumulative returns will grow and vice versa. If the returns are distributed evenly between positive and negative then the cumulative returns will be stagnant.

Rescaled Range (RSR)

From the vector of cumulative deviations, a range is computed by subtracting the minimum of the cumulative deviation from the maximum. Then this range is divided by the standard deviation to get the rescaled range.

$$RSR_1 = \frac{[cd_{\max} - cd_{\min}]}{\sigma_1} \qquad (8)$$

Where RSR = rescaled range

Several values are required to fit a straight line by the method of least squares and to compute the slope of a line through regression. Therefore to get more data the same cumulative deviation vector is divided into two equal halves and applying the same rescaled range procedure applied above another two rescaled ranges are computed. The average is taken as the second rescaled range.

$$RSR_2 = \frac{\frac{[cd_{\max} - cd_{\min}]}{\sigma_{1/2}} + \frac{[cd_{\max} - cd_{\min}]}{\sigma_{1/2}}}{2} \qquad (9)$$

RSR_2 = (rescaled range of first half + rescaled range of the second half)/2

If the number of returns is in 2^n then the rescaled range could be applied efficiently as the returns could be recursively scaled in equal halves. If not after some iterations it will go out of proportion. Recursively several non-overlapping segments of equal length of cumulative returns are taken and their average rescaled ranges are computed.

$$RSR_n = \frac{\frac{[cd_{\max} - cd_{\min}]}{\sigma_{1/n}} + *** + \frac{[cd_{\max} - cd_{\min}]}{\sigma_{1/n}}}{n} \qquad (10)$$

After several iterations two vectors of values emerge. The first vector is the dependent variable, the rescaled range of returns and the second vector is the number of days whose returns are included in every segment. Then the logarithms of both vectors are computed.

Column Vector 1		Column Vector 2	
RSR (dependent variable)	Ln log RSR	Days included in every segment (independent variable)	Ln log Days
RSR_1	Ln log (RSR_i)	t	Ln log (t_i)
*	*	*	*
*	*	*	*
*	*	*	*
RSR_n	Ln log (RSR_n)	*	Ln log (t_n)

With the natural logarithmic vectors (RSR and t), regression coefficient is computed taking ln log rescaled range returns as the dependent variable and the ln log number of days as independent variable. The resultant regression slope coefficient is the Chaotic Exponent.

$$y = \alpha t^{\beta} \qquad (11)$$

$$\text{Ln log } Y = \alpha + \beta \ln \log t \qquad (12)$$

$$\text{Ln log (RSR)} = \alpha + \beta \ln \log (t) \qquad (13)$$

$$CE = \beta \qquad (14)$$

$$r = \alpha * t^{CE} \qquad (15)$$

$$\begin{cases} Returns\ are\ pure\ white\ noise\ if\ = \alpha * t^{0.45\ to\ 0.55} \\ Returns\ are\ mean\ reversing\ if\ = \alpha * t^{<0.45} \\ Returns\ are\ persistent\ if\ = \alpha * t^{>0.55} \end{cases} \qquad (16)$$

Data

Nine shares traded in Kuala Lumpur Stock Exchange (KLSE) were selected at random and their closing prices were downloaded from Yahoo finance website, for ten years from January 2000 to December 2009. There were 2450 closing prices excluding the holidays for this period of ten years. The prices relating to the last four years i.e. from January 2006 to December 2009 were considered for short-term (one year) CE calculations. For long term CE, out of 2450 prices recent 2048 prices were considered and the remaining data were discarded. This is to satisfy the 2^n rule of power law. The CE could be effectively calculated if the data is in the powers of two as the rescale range requires the data to be recursively divided into two equal halves.

Share market Indices

The stock market index is another FTS which is a weighted average price of all shares traded in a stock exchange. The unique feature of this index is that it combines all positive price movements and negative price movements in a day and hence there is a smoothing effect. The principal difference between the share price and the stock index is that the stock index is the weighted average of all shares. It represents all macro economic variables and hence there will be no company specific factor could influence it. But the individual company share price will be influenced by company specific factors. The indices of KLSE, Germany, Australia, UK, Singapore, NIKKE and US indices Dow Jones, NASDAQ and Std & Poor were downloaded from Yahoo finance and the same procedure applied for share prices is applied to calculate CE of indices.

Exchange rates

Exchange rates of nine currencies in relation to Ringgit Malaysia were downloaded from Pacific Exchange rate services web site. The currencies chosen are stronger than Ringgit meaning that for one unit of foreign currency we have to pay more than one Ringgit. The soft currencies were left out due to their larger volatility and instability. The US dollar was omitted because the US dollar was pegged to Ringgit at RM 3.80 till July 2005 and after depegged. The USD exchange rate therefore gives non continuous extreme data is an outlier hence omitted from analysis. To calculate CE for exchange rates the same procedure adopted in shares and indices is applied here. The main difference between share price and the exchange rates is the countries involved. Share price movements are influenced by company specific factors and country specific factors, the indices are influenced only by macro economic factors pertaining to the specific country but foreign exchange rates are sensitive to the relative macro economic variables of host and home countries (Diebold 1990). Therefore the CEs of indices and exchange rates will be more consistent and normally distributed when compared with share prices.

Results and Discussion

The long-term CEs and short-term CEs have been computed by using the specific years' closing prices data. The selected shares' CEs are given below.

Table 1. Chaotic Exponents of selected shares

Company	2006	2007	2008	2009	10 years
OSK	0.443**	0.677*	0.390**	0.527	0.723*
BERJAYA TOTO	0.558*	0.601*	0.538	0.272**	0.686*
IOIPROP	0.345**	0.569*	0.497	0.376**	0.613*
OYL	0.489	0.674*	0.512	0.404**	0.560*
YTLPOWER	0.594*	0.332**	0.412**	0.587*	0.543
IOI PROP	0.284**	0.442**	0.183**	0.597*	0.503
ANN JOO	0.263**	0.521	0.416**	0.781*	0.471
TYLCORP	0.418**	0.580*	0.233**	0.201**	0.431**
JASA TIASA	0.290**	0.344**	0.629*	0.743*	0.425**

* CEs more than 0.55 indicate persistent behaviour
** CEs less than 0.45 indicate anti-persistent behaviour
CEs between 0.45 and 0.55 indicate stochastic behaviour

The long-term CEs for different companies are sorted in descending order. Four companies show persistent behaviour, three companies show stochastic behaviour and two companies exhibit anti-persistent behaviour. Among the share CEs there is no uniformity in the long run. If we look short-term CEs same mixed pattern is observed. In 2006 six companies show anti-persistent behaviour, two companies show persistent behaviour. Only one company show random behaviour. Similar mixed pattern could be seen in 2007, 2008 and 2009. When we compare the results company-wise the same inconsistent behaviour is observed. None of the companies show uniform behaviour. IOI properties and TYL corporation show anti-persistent behaviour in three years. All other companies show mixed patterns over the years. These results imply two things. Firstly, the share returns are not truly random as they do not show any consistency either in the short-run or in the long-run. As pointed out by earlier researchers through different statistical tests that the share returns do not exhibit pure normality. There is fat tail, skewness and kurtosis which make the forecasting relevant. The chartist approach also advocates that the share returns are not absolutely normally distributed. Secondly, the market is not so efficient because most of the company share prices either show persistent or anti-persistent behaviour. Hence forecasting and hedging are relevant in KLSE.

Table 2. Chaotic Exponents of selected indices

Indices	2006	2007	2008	2009	10 years
KLSE	0.439**	0.726*	0.724*	0.548	0.605*
Australia	0.639*	0.596*	0.575*	0.196**	0.567*
Germany	0.614*	0.421**	0.553*	0.429**	0.530
NASDAQ	0.574*	0.428**	0.48	0.445**	0.515
Singapore	0.392**	0.524	0.412**	0.606*	0.494
NIKKE	0.738*	0.336**	0.400**	0.493	0.472
Std & Poor	0.449**	0.331**	0.533	0.414**	0.446**
UK	0.563*	0.344**	0.435**	0.236**	0.409**
Dow Jones	0.318**	0.294**	0.532	0.414**	0.402**

* CEs more than 0.55 indicate persistent behaviour
** CEs less than 0.45 indicate anti-persistent behaviour
CEs between 0.45 and 0.55 indicate stochastic behaviour

The long term CEs for different indices have been sorted in the descending order and reported. KLSE and Australian indices show persistent behaviour, Std & poor, UK and Dow Jones indices show anti-persistent behaviour and the remaining indices exhibit stochastic behaviour. Short term indices also show inconsistency. Out of 36 short-term CEs calculated over a four year period 11 times the indices show persistent behaviour, 19 times anti-persistent behaviour and six times stochastic behaviour. Short-term country wise analysis show anti persistent behaviour for Dow, UK and Std & Poor. Even in these indices one year index is not anti persistent. KLSE, Germany, Singapore NASDAQ and NIKKE show mixed CEs. Australia shows persistent behaviour. Contrary to our expectations even the indices show mixed pattern. It implies that even weighted average of prices (indices) in the stock market do not remove the random behaviour. Even the advanced countries' stock market indices do not show consistent behaviour. The indices also behave like shares and there is no uniqueness in indices.

Table 3. Chaotic Exponents of selected exchange rates

Currency	2006	2007	2008	2009	10 years
KRW	0.642*	0.605*	0.483	0.503	0.582*
NZD	0.634*	0.612*	0.363**	0.415**	0.544
GBP	0.455	0.617*	0.416**	0.522	0.501
EUR	0.562*	0.530	0.376**	0.283**	0.460
JPY	0.454	0.358**	0.444**	0.454	0.458
CHF	0.538	0.513	0.371**	0.345**	0.430**
CAD	0.310**	0.491	0.455	0.456	0.428**
AUD	0.437**	0.598*	0.427**	0.205**	0.411**
MXN	0.602*	0.417**	0.174**	0.455	0.365**

* CEs more than 0.55 indicate persistent behaviour
** CEs less than 0.45 indicate anti-persistent behaviour
CEs between 0.45 and 0.55 indicate stochastic behaviour

The CEs of exchange rates also behave similar to shares and indices. More anti persistent behaviour could be seen over the years. Out of 36 indices in total 15 indices show anti persistent behaviour in the study period. Eight indices show persistent behaviour and 13 exchange rates show stochastic behaviour. The long-run CEs results are also mixed. Four exchange rates show anti-persistent behaviour, one persistent behaviour and another four currencies show stochastic behaviour. Currency wise analysis results also show anti-persistent behaviour by Australian Dollar, stochastic behaviour by Canadian Dollar and the other currencies show a mixed pattern. Since 23 CEs are not stochastic we could confirm even the exchange rate market is not purely Brownian meaning that the distributions are not purely normal. It may be somewhat skewed and may be with fat tails.

Conclusion

The financial time series of different categories show different chaotic exponents and they are not consistent. Both the short-term and long-term chaotic exponents show mixed pattern and they are not consistently close to 0.5 as expected by efficient market theory. Majority of FTS returns show either persistent or anti-persistent behaviour, only a few FTS returns show stochastic behaviour. This clearly shows that there is non randomness and fat tails in FTS and hence the forecasting and hedging decisions are relevant.

The Markovian principle advocates that there is no memory in FTS returns and the current prices dependent only on the previous price. No one can predict the future returns with the help of past data are effectively nullified from our results. The time series analysed above are not purely Brownian (Weiner process) motions, there is some sort of non normality. Our findings are important which support the chartist approach of forecasting of time series as appropriate. The Gaussian distribution assumption widely applied in Value at Risk (VaR) calculations, ARIMA and GARCH modeling and pricing of derivatives under Black and Scholes model are approximations and are not accurate. This approximation may lead to mispricing of derivatives.

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